Part 1: Introduction and History

Art is often a representation of an individual or culture's identity and history. It is a reflection or lens in which the artist viewed, lived, and valued their world. Many cultures have used art as a means of expression and communication that helped to perpetuate their values. The traditional Hawaiian hand-tapped tattoo is one way the ancient Hawaiians used art in their culture. The process of getting one of these tattoos starts with multiple protocols and prayers and is considered a sacred rite more than an artwork. Each of these tattoos is unique. The design bestowed is based on the wearer's role in the community and genealogy. Most of the Hawaiian tattoo motifs utilize diamonds, triangles, and curved shapes. Also, the negatives of these tattoos form unique geometric patterns. The one thing in common with all of the Hawaiian tattoos is that they contain some type of symmetry.

Students will draw an image that reflects their culture to help them grasp the concept of definite integrals and area between the curves. Having students draw their own images will make math more personal to them and more relatable. It also allows the students to show their creativity with math, which otherwise is uncommon.

Part 2: Goal of Lesson Plan

The goal of this project is to mix the two abstracts, art and mathematics, to give the students, who are learning calculus, a better transition into differentiation and integration, the basis of calculus. The following Mathematics Common Core State Standards are utilized:

◊ MAC.9.6: Use Riemann sums, the trapezoidal rule, and technology to approximate definite integrals of functions represented algebraically, geometrically, or by tables of values

◊ MAC.9.7: Find specific antiderivatives using initial conditions, including finding velocity functions from acceleration functions, finding position functions from velocity functions, and applications to motion along a line

\[
x^2 + (y - 2.75)^2 = 2^2
\]

\[
f(x) = -5 \cdot \log(-x + 4) + 5
\]

\[
\frac{(x + 1)^2}{1^2} + \frac{(y - 2.75)^2}{(1.25)^2} = 1
\]

Written by Mitchell Walker
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◊ MAC.9.8: Use definite integrals to find the area between a curve and the x-axis, the average value of a function over a closed interval, and the volume of a solid with known cross-sectional area.

◊ MAC.10.5: Apply the fundamental theorem of calculus; i.e., interpret a definite integral of the rate of change of a quantity over an interval as the change of the quantity over the interval, that is

\[ \int_a^b f'(x) \, dx = f(b) - f(a) \]

◊ MAC.10.6: The student: Uses the following properties of definite integrals to evaluate definite integrals:

\[ \int_a^b [f(x) + g(x)] \, dx = \int_a^b f(x) \, dx + \int_a^b g(x) \, dx \]

\[ \int_a^b k \cdot f(x) \, dx = k \int_a^b f(x) \, dx \]

\[ \int_a^a f(x) \, dx = 0 \]

\[ \int_a^b f(x) \, dx = -\int_b^a f(x) \, dx \]

\[ \int_a^b f(x) \, dx + \int_b^c f(x) \, dx = \int_a^c f(x) \, dx \]

If \( f(x) \leq g(x) \) on \([a, b]\), then \( \int_a^b f(x) \, dx \leq \int_a^b g(x) \, dx \)

◊ MAC.10.10: Find average and instantaneous rates of change.

Part 3: Methodology

◊ Practice with a few basic combinations of functions to form obvious images/symbols (each of which the students can choose, that increase in complexity as they progress).

◊ Students should choose images that reflect their background/interests.

◊ Draw image free hand.

◊ Draw image on graph paper, plot the image on paper, using functions to describe every mark made (Must incorporate trig functions, polynomials, and logarithmic functions).

◊ Find the derivatives of 10 of the functions used.

◊ Approximate the area of the image’s silhouette using Riemann Sum and trapezoidal rule.

◊ Now find the actual area of your image’s silhouette using definite integrals.

First, to get the students comfortable with creating images with functions, get them to graph their initials or a single letter. You may want to do a slight review on finding lines of best fit with the students.

Remember that the students final graphs should be drawn free handed, but they can use online tools/calculators to play with which functions to use.

In this example, I used two functions to create the right side of an M and then I reflected those about the y axis to create the other half.

This is the M with the restrictions on the domain.
Next the students should create something slightly more advanced, mixing line functions with a polynomial, to make a very simple image.

Then the students should add restrictions on the domains of the functions they’ve chosen so that the image is clear.

Now the students should try something a bit more challenging like a face, symbol, etc. In this example I used three quadratics to create part of a dog.

I added in the restrictions for the first three functions where they equal each other. To keep the image symmetric, I took the function \( f(x) = 10(x-2)^2 - 2 \) and shifted it to the left by 4.

Adding in the restrictions where the previously added in function meets the other functions, we get the following:

To make the image more defined, I added in conics for the eyes, nose, and tongue.

Adding in restrictions for the nose and tongue, we get the following:

When the students are done they can color in their image to make it more aesthetically pleasing, but since it’s just for practice the students should only add detail if they want to.

Now that the students have a better idea of how this works, they will work on their final image (This is the one that they will be using calculus to find the area). This one must incorporate logarithmic functions, trigonometric functions, and polynomials. The functions will be the ones that the students will be differentiating and integrating, so make sure all of the functions used are written down and organized.

**Reminder:** the students final graphs should be drawn free handed, but they can use online tools/calculators to play with which functions to use.
Here is an example of how the students should set it up. This image is the setup of the top of a honu.

Next I merged the Body and the legs of the honu (both with restrictions).

Notice the feet seem to be going the opposite way we’d like them to. To make the honu more realistic I reflected all of the feet functions with respect to the x-axis and shifted them.

Here are all the functions used (without restrictions).

Now the Students should break the silhouette of their image up into segments where the top and bottom functions differ.

Then the students will need to find the area of each segment, using Riemann Sum, trapezoidal rule, and definite integrals. Finally, to find the total area of their image, the students will need to do is find the sum of each segments area.

Overall, the students should:

◊ Have three versions of their image, their first drawing, graph and functions without restrictions, and the graph with functions and restrictions broken up into segments, all to be done by hand. The students may use their graphing calculators to find the functions they want to use.

◊ Choose 10 of their functions and tell whether or not the function is concave up and/or down on the interval of the image (must include at least one trigonometric and logarithmic function).

◊ Find the area of their image using both the Riemann sum and trapezoidal rule.

◊ Find the area of the silhouette of their image using definite integrals. Students may use technology to double check their results.

Part 4: Conclusion

Calculus is a milestone in any student’s educational career. This lesson plan will, hopefully, give students a smooth transition from trig/pre-calculus into the concepts of calculus. This lesson plan is meant to follow the student throughout their first semester calculus class so that they aren’t just looking at the functions and graphs in their textbook, but functions and graphs that they came up with.