How can various aspects of kalo production be modeled with systems of linear equations?

by Froilan Garma

Standard Benchmarks and Values:

Mathematics Common Core State Standards (CCSS):

• A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

• A.REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.

Hawai‘i Content & Performance Standards (HCPS) III:

• MA.AI.10.5: Solve systems of two linear equations in two variables algebraically and graphically.

Nā Honua Mauli Ola (NHMO) Cultural Pathways:

• ‘Ike Pilina (Relationship Pathway): Nurturing respectful and responsible relationships that connect us to akua, ʻāina and each other through the sharing of history, genealogy, language and culture.

• ‘Ike Mauli Lāhui (Cultural Identity Pathway): Perpetuating Native Hawaiian cultural identity through practices that strengthen knowledge of language,
culture and genealogical connections to akua, ‘āina and kanaka.

- 'Ike Honua (Sense of Place Pathway): Demonstrating a strong sense of place, including a commitment to preserve the delicate balance of life and protect it for generations to come.

Enduring Understandings:

- Lo‘i are an integral part of Hawai‘i’s history and culture.
- The modern day economy provides lo‘i business opportunities.
- Apply addition and substitution method to Hawaiian lo‘i system situations.

Background/Historical Context:

Although people of Hawai‘i now import more food than they produce, Hawaiian planters (in the days before European contact) were once able to cultivate enough food to sustain a population of at least 300,000 people. Captain James Cook and other early visitors remarked about Hawaiians’ intelligent use of land and water, their abundance of food produced, and their advanced knowledge of resource conservation. The average maka‘āinana (commoner) was an agriculturist at heart, as evidenced in the many Hawaiian myths, traditions, and folk customs that are more concerned with plant than marine life (which merely supplemented one’s diet, at the time) and in the dedication of boys to Lono, the god of agriculture (as opposed to Kū, the god of war).

Nowhere else in the world was kalo (taro or Colocasia esculenta) cultivated so skillfully or intensively as in Hawai‘i. Perhaps this was due in part to Hawaiians’ belief that kalo was the older sibling of the first man Hāloa and, ever afterward, shared a mutualistic relationship with Hāloa’s Hawaiian descendants. The planters of wetland kalo possessed the skills of engineers, building earthen walls to enclose each lo‘i kalo (taro patch) and creating ‘auwai (watercourses) to conduct cold water from springs or principal streams. Openings within the walls were used to control the flow of water and nutrients (as well as the relative density of the mud) from the upland-most to the lower lo‘i. Varieties of dryland kalo were also produced but required less effort, flourishing within the clearings of forests.

The Hawaiians were also unique among Polynesians in preferring poi (cooked and pounded taro corm) as their staple food. Mature plants were harvested by hand and had their leaves removed, were washed and then steamed in an imu (earthen oven) until tender, were peeled and scraped, and finally were pounded upon a pāpā ku‘i ‘ai (wooden poi-pounding board) with a pōhaku ku‘i ‘ai (stone poi pounder). Today, poi and pa‘i ‘ai (a thicker, less processed kalo product) are being produced and sold by local lo‘i farmers and are available at Hawaiian restaurants and supermarkets, giving kalo the potential to once again serve as a highly viable, sustainable, local agricultural product.
Authentic Performance Task:

In small groups (or individually), students will visit a local lo‘i and, based on researched information, write short responses for the following:

1. What is a system of linear equations?

2. When is it easier to use the substitution method instead of the addition method for solving systems of linear equations?

3. What are break even points?

4. What are some real world applications of systems of equations?

5. How can kalo production be modeled as a system of linear equations?

   In the same groups, students create a system of linear equations and decide whether or not the created system has one, infinitely many, or no solutions.

Authentic Audience:

The school community, including (but not limited to) teachers, students, and students’ family members.

Learning Plan:

1. Discuss the history and significance of kalo cultivation in Hawai‘i and have students suggest algebraic applications within lo‘i.

2. Define and explain the following for students: equation, linear equation, system of equations, and finally system of linear equations.

3. Provide the substitution "recipe" (method) for finding the solution of a system of linear equations:

   i. Pick one of the two equations and isolate one of its variables (i.e., get it alone on one side of the equation).

   ii. Substitute the value of the isolated variable for the same variable in the other equation (in order to create a single-variable equation).

   iii. Solve for the remaining variable.

   iv. Substitute the numerical value of the defined variable (from part iii) for its respective variable in the either of the equations in order to solve for the other variable.

   v. Check for accuracy by substituting the former variable’s value into the other equation and solve for the remaining variable (which should have the same value as the latter variable in part iv).

4. Demonstrate an example:

   Problem: You have 280 total lbs of kalo products, and your lo‘i business needs at least $2,000 to break even. Poi is sold for $5 per 1-lb bag, and pa‘i ‘ai is sold at $5 per ½-lb container. How many pounds of each kalo product could you sell in order to break even?

   Consider the following system: Let x be bags of poi and y be containers of pa‘i ‘ai.

   \[
   \begin{align*}
   x + 0.5y &= 280 \\
   5x + 5y &= 2000
   \end{align*}
   \]

   Solution: In this problem we will be following the recipe for the substitution method. Starting with the first equation, we will isolate x:

   \[
   \begin{align*}
   x &= 280 - 0.5y
   \end{align*}
   \]
Next we will create an equation with only one variable. This is the substitution part:

\[ 5(280 - 0.5y) + 5y = 2000 \]

Now we simplify, and solve for the single variable, which in this case is \( y \):

\[
\begin{align*}
1400 - 2.5y + 5y &= 2000 \\
1400 + 2.5y &= 2000 \\
1400 &= 2000 - 2.5y \\
-600 &= 2.5y \\
\frac{-600}{-2.5} &= y \\
240 &= y 
\end{align*}
\]

Now knowing that we need to sell 240 containers (or 120 lbs) of pa’i ʻai, we will substitute our known variable \( y \) into the other equation. (Give students the option of choosing the equation in which they should substitute the variable.) In the event they choose the first equation:

\[
\begin{align*}
x + 0.5(240) &= 280 \\
x &= 280 - 120 \\
x &= 160 
\end{align*}
\]

Therefore, you might need to sell 160 lbs (or 160 bags) of poi and 120 lbs (or 240 containers) of pa’i ʻai in order to break even.

5. Provide the addition “recipe” (method) for finding the solution of a system of linear equations:

a. Rewrite both equations in the standard form of \( Ax + By = C \). (If already done, then skip this step.)

b. Multiply either or both equations by non-zero integers in order to make the sum of their \( x \) coefficients (or \( y \) coefficients equal to zero).

c. Add the two equations together.

d. Solve for either \( x \) or \( y \).

e. Solve for the other variable (to check work).

6. Show an example:

Consider the following system:

\[
\begin{align*}
6x - 8y &= 22 \\
-6x + 4y &= -14 
\end{align*}
\]

For this particular set, show students that the \( x \) terms cancel out. They are inverses of each other, so when these terms are added, they cancel out.

\[
\begin{align*}
6x - 8y &= 22 \\
-6x + 4y &= -14 \\
We simply add the two equations. \\
\hline
-4y &= 8 \\
y &= -2 
\end{align*}
\]

The take away for this example is to show students that terms can be eliminated by adding the two equations. Addition is similar to the addition they are familiar with.
7. Provide a real-world example:

Problem: Suppose you sell a total of 500 lbs of kalo products to a local market. A bag of poi costs $5, whereas a \( \frac{1}{2} \)-lb container of pa‘i ‘ai costs $5. The market collects $2700 in sales. Find the number of pounds of each type of kalo product sold.

Solution: When \( x = \) poi and \( y = \) pa‘i ‘ai, the situation can be modeled with the following system of equations:

\[
\begin{align*}
x + 0.5y &= 500 \\
5x + 5y &= 2700
\end{align*}
\]

(Let \( x \) be bags of poi and \( y \) be containers of pa‘i ‘ai.)

\[
\begin{align*}
x + 0.5p &= 500 \\
(-0.1)(5x + 5y) &= (-0.1)(2700)
\end{align*}
\]

Multiply both sides of the second equation by -0.1 in order to eliminate the \( y \) variable.

\[
\begin{align*}
x + 0.5y &= 500 \\
-0.5x - 0.5y &= -270
\end{align*}
\]

Add the two equations

\[
0.5x + 0 = 230
\]

Sum of the two equations

\[
x = 230/0.5 \\
x = 460
\]

Isolate \( x \)

\[
x + 0.5y = 500 \\
460 + 0.5y = 500
\]

Substitute \( x \) into the original equation.

\[
0.5y = 500 - 460 \\
0.5y = 40 \\
y = 40/0.5 \\
y = 80
\]

Subtract 460 from both sides of the equation. Then, divide both sides by 0.5

The ordered pair is given as \((460, 80)\), where 460 lbs (or 460 bags) of poi and 80 lbs (or 160 containers) of pa‘i ‘ai were sold.
8. Reflection: When would it be easier to use the addition method instead of the substitution method?

9. Show an example of a system with no solution:

\[
\begin{align*}
6x + 8y &= 10 \\
3x + 4y &= 8 \\
6x + 8y &= 10 \\
3x + 4y &= 8 \\
-6x - 8y &= -16
\end{align*}
\]

Multiply by -2
Add.

This system implies that \(0 = -4\), which is a false statement. Therefore, this solution is an empty set. Graphing this system of equations would show parallel lines, and parallel lines do not intersect.

To prove this point, have students find the slope of each line. Ask them:
- What does this mean?
- If the solution to the system is inconsistent, then what must be true about the system’s graphs?

10. Show an example of a system with many solutions. Pick a point outside of one of the aforementioned lines. Ask students why this point is not a “solution” to this equation even though it says “infinitely many solutions.”

11. Have students complete the following practice exercises:

a. Assuming that the following system of equations models the potential amounts of wetland kalo (x) and dryland kalo (y) produced within a lo‘i, solve for the amount of each:

\[
\begin{align*}
12x + 11y &= 892 \\
-x + y &= 15
\end{align*}
\]

b. A locally-owned company that farms kalo has a fixed cost of $19,000. It costs $2 to produce each kalo corm. The selling price per corm is $8. (Hint: let “x” represent the number of corms produced and sold.)

i. Create a system of equations for this problem.

ii. How much kalo will the company need to sell to break-even?
c. The figure below shows a graph of the revenue and cost functions for a lo‘i that sells kalo. The owner sells a box of kalo corm for $50 each, the cost of which to grow is $30. Using the information in the graph, answer the following questions.

i. How much kalo must be produced and sold for this kalo company to break even?

ii. How much kalo must be sold to make a profit?

![Profit and Loss Graph](image)

\[ C(x) = 10000 + 30x \]
\[ R(x) = 50x \]

Amount of Kalo Corms Produced and Sold

Dollars

0
100
200
300
400
500
600
700

Kalo Production

![Kalo Production Bar Chart](image)

<table>
<thead>
<tr>
<th>Year</th>
<th>Amount of Kalo Produced (thousands of lbs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2008</td>
<td>5.2</td>
</tr>
<tr>
<td>2010</td>
<td>6</td>
</tr>
<tr>
<td>2012</td>
<td>6.8</td>
</tr>
<tr>
<td>2014</td>
<td>7.67</td>
</tr>
</tbody>
</table>

Kimo and Bradley both own lo‘i on opposite sides of Kaua‘i. Kimo’s ahupua‘a has been getting a lot of rain, and Kimo has been spending extra time on the field. Bradley has been combating an invasive plant species growing in his lo‘i that is affecting his kalo’s growth. Based on the following bar graph showing kalo production from 2008 to 2014 (Kimo's in dark blue, Bradley's in light blue), create a system of equations.
e. Colin’s lo’i has been producing less kalo than in past years. Recently, he found out that water is being diverted from the kahawai or stream that feeds his lo’i. However, his friend Stacy’s lo’i has been increasingly productive. Based on the following system of equations (modeling the situation), when might Colin’s and Stacy’s lo’i produce the same amount of kalo?

\[
\begin{align*}
x + 2y &= 50 \\
-x + 2y &= 24
\end{align*}
\]

f. If you wanted to grow kalo and another crop (e.g., mai’a or banana) on your 240-acre farm, and you wanted to plant 80 more acres of kalo than the other crop, how many acres of each crop do you need to plant? (Write a system of equations that could model this situation.)

12. In small groups (or individually), have students visit a local lo’i and, based on researched information, respond to the following:

a. What is a system of linear equations?

b. When is it easier to use the substitution method instead of the addition method for solving systems of linear equations?

c. What are break even points?

d. How can kalo production be modeled as a system of linear equations?

e. What are some real world applications of systems of equations?

In the same groups, have students create a system of linear equations and decide whether or not the created system has one, infinitely many, or no solutions.
References:


