
Photo from Kumulau.com

Standard Benchmarks and Values

Common Core State Standards for Mathematics:

1. **A.REI.1 -** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. **F.IF.7 -** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

3. **F.TF.7 -** Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Na Honua Mauli Ola Pathways:

1. **NHMO.1** – Incorporate cultural traditions, language, history, and values in meaningful holistic prcsesses to nourish the emotional, physical, mental/intellectual, social, and spiritual well-being of the learning community that promote healthy mauli and mana.

2. **NHMO.3** – Sustain respect for the integrity of one's own cultural knowledge and provide meaningful opportunities to make new connections among other knowledge systems.

TRIGONOMETRIC SLIDING SPEARS

Geuseppe Ayala

11th - 12th Grade
6 - 7 class periods

How can the use of trigonometric properties allow students to solve for unknown quantities?

- How can the use of trigonometric properties allow students to solve for unknown quantities?
- What can be inferred from the data about the methods that the students used to throw the ihe and the methods the students used to obtain the desired measurements and calculations?

Enduring Understandings

- Students will apply reasoning and problem solving in order to solve real-world mathematical problems in a culturally relevant context.
- Students will observe, model, and solve mathematical problems that exist in a physical game-based scenario.
- Students will be able to obtain and manipulate data in order to draw conclusions of the given situation.
- Students will be able to think critically and logically to approach and solve mathematical problems.
- Students will be able to cooperate with other students in order to solve mathematical problems.

Timeframe for Implementation

Approximately six to seven class periods. These will consist of

- Two to three class periods as review (trigonometric identities/properties, linear transformations, slope, and plotting data points to create best fit lines).
- One class period to go over the cultural aspects of the lesson (history of Makahiki, Hawaiian Anthropic Units) and unit conversion.
- One class period to go into the field and perform the lesson plan.
- One to two class period for sharing and discussion.

Preparation time for the lesson is approximately two to three hours. Price range is about $20 - $30 but may vary depending on the number of students and the instructors choice of materials.

BACKGROUND

Critical Skills and Concepts

- Use problem solving to solve for unknown quantities.
- Understand how trigonometric functions can be used to measure distances and angles.
- Formulate and use unit conversions for measuring distances.
- Be able to plot data and infer information about the data.
- Be able to draw conclusions about the situation using the data.

Background Information

In the ancient Hawaiian culture, games and sport were integral parts of everyday life. These ancient games, called pa'ani kahiko were numerous and popular.

These games had an even greater importance during the yearly makahiki festival which spanned the months of October – February or November – March (this all depended on the year).

This time was meant for the arrival of the god Lono and the setting aside of the god Ku. This meant that warfare comes to a complete halt and the Hawaiian people are allowed to come together, procreate, and grow anew.

The games that were played varied, and were played for different reasons. While these games were meant to be used for leisure, they were also opportunities for Hawaiian warriors to show physical prowess and skill. Some of these games included wrestling, sledding, surfing, along with disk, dart, and spear throwing. There were also mental games like Konane (the Hawaiian form of checkers) and riddles.

Hawaiian Anthropic Units were used by ancient Hawaiians as measurements. Hawaiian Anthropic Unit measurements do not have a length that is universally accepted. This is because the Units are based off of one's own body measurements; hence a measurement for one person may be smaller or larger than another person's. These measurements were used in a variety of situations, such as navigating and distance approximation.

The Hawaiian Anthropic Units are as follows:

1. ***'owā*** = ½ finger's width
2. ***mākahi*** = 1 finger's width
3. ***mālua*** = 2 fingers' width
4. ***mākolu*** = 3 fingers' width
5. ***māhā*** = 4 fingers' width
6. ***kīko'o*** = tip of pointer finger to tip of thumb (hypotenuse)
7. ***pī'ā*** = length of hand (tip of middle finger to heel of hand)
8. ***ha'ilima*** = elbow to fingertips
9. ***iwilei*** = collarbone to fingertips
10. ***muku*** = other elbow to fingertips
11. ***anana*** = wingspan

LESSON

- Students will create tables and graphs for the data that they collected.
- Students will then share to the class or in their groups how the change of the independent variable affected the dependent variable and compare their data with their classmates/group-mates to find out who was the most skilled at throwing spears.
- Students will share to the class a few examples of how they calculated some of the quantities using trigonometric functions.
- Students will share to the class how they formed their formula's for unit conversions.

Learning Plan - Teacher

1. Preparation: Obtain the following:

- Fun Noodle Swimming Aids (These will act as the ihe)
- Fillers
- 30 ft. Rope
- Protractors
- Goal Post

Authentic Audience

- Students and their friends
- Parents
- Community

Notes

1. When referring to Fillers, these are anything the instructor can think of to put inside of the Fun Noodle. This can be anything from pieces of wood, recycled paper, etc. This is needed due to the weight of the Fun Noodle itself. Without it, the Noodle would be too light and would deviate too easily when thrown and when sliding.
2. When referring to Goal Posts, these are objects that can be placed on the ground and will remain there for the entirety of the lesson. These will be the posts in which the students will attempt to throw the ihe. This can be anything from cones, sticks, tape (if you are doing this on a hard floor). The instructor will need four pairs of goal posts to mark off the distances the students will throw the ihe through
3. The amount of Fun Noodles, rope, and protractors all depend on how many groups the instructor will divide his/her groups into. One Fun Noodle, two 30 ft. ropes, and a protractor should be given to each group.

2. Measure the appropriate Hawaiian Anthropic Units listed in the Background/Historical Context section and record them. Make sure to record the unit conversions.

Example: 1 Haʻilima = 3 Kikoʻo = 1.5 ft.

3. Choose desired Hawaii Anthropic Units and mark them down using different colored markers on each 30 ft. rope.

Notes

Since Hawaiian Anthropic Units vary for each person, the instructor will measure their own measurements. These will be used as the Hawaiian Anthropic Units in this lesson.

LESSON

In the Classroom (Before Activity)

Give brief reviews and introductions to the following:

- Trigonometric properties and identities
- Plotting data and best fit lines
- The Makahiki. This means its context, importance, the events that occurred, the games that were played and their importance
    - It is important to mention how the Hawaiians used this game as a test of skill. This will be the hook.
- Units, and unit conversions
- Hawaiian Anthropic Units.

On the Field

1. Divide the students into equal groups.
2. Give each group an ihe, protractor, a pair of 30 ft. rope
3. Select four distances to place the Goal posts.

    Example: 15 Ha‘ilima, 20 Ha‘ilima, 25 Ha‘ilima, 30 Ha‘ilima

In the Classroom (After Activity)

1. Give the students the unit conversions.
2. When students have created their spreadsheets (including data, and a graph best fit line(s)) display each group's graph to the class. Have the student's judge, based on the graph, which group was more skilled at throwing spears.

Following the Activity

Have the students consider the following questions:

1. Given that the graph is Angle vs. Distance, what does the slope of each line say about the skill of the thrower?
2. What role does the y-intercept have in judging the skill of the thrower?
3. What if the best fit line has a low slope but a high y-intercept?
4. What would the graph of a very skilled spear thrower look like?

Have the students consider these other questions:

1. If there was any difference between the measured angle and the calculated angle, what are the reasons for the difference?
2. Do the measured angles and the calculated angles trend the same way? (ie. Do they both increase/decrease at the same time?)
3. Do you get the same calculated angle change if you calculate using Hawaiian Anthropic Units? Why or why not?
4. What possible errors could have occurred during the lesson? How do these errors affect the data?

Students

Set up on the Field:

1. Pick a designated starting point.
2. Place one 30 ft. rope on the around. Make sure that one of the ends of the rope is at the designated starting point. Call for the instructor to secure the rope into the ground and place the goal posts. This will be called the reference rope.

Activity

1. Throw the ihe underhand toward the first set of goal posts. The ihe may be thrown in any way
2. Once the ihe stops moving students will use the other 30 ft. rope to measure the distance from the starting point to the tip of the ihe. Record the data.
3. Use any method (example. Inverse trig functions, Law of Cosines, Law of Sines) to calculate the angle between the trajectory of the ihe and the reference rope. If any other data is needed in order to calculate the angle, state what is being measured and record it
4. Use the protractor to measure the angle trajectory of the ihe and the reference rope
5. Repeat for each of the given goal post.

Notes

Recall that the other 30 ft. rope will be marked in Hawaiian Anthropic Units. Thus, data will be in Hawaiian Anthropic Units.

Student calculating angles.

Photo by Valerie Dao

After the Activity

1. Convert the distances from Hawaiian Anthropic Units to English units.
2. Calculate the angle in between the trajectory of the ihe and the reference rope using desired method.
3. Create a Microsoft Excel spreadsheet. Create tables displaying the distances (in both Hawaiian Anthropic Units and English Units), and the calculated angle. Also include graph of Angle vs. Distance, the graph must include a best fit line for the data.
4. Include the slope of the best fit line into the data.
   a. If the slope of the best fit line is negative, take the absolute value of the slope and record it.

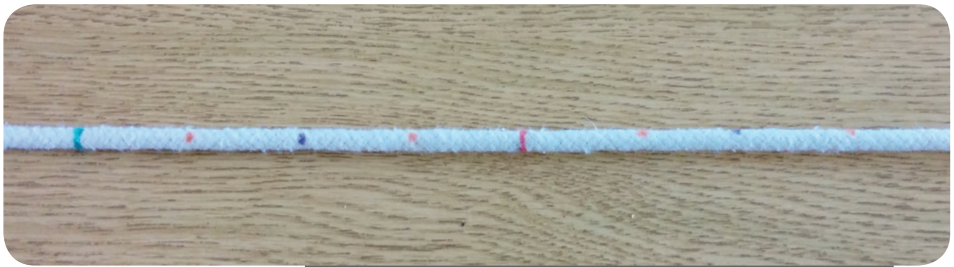
In the Classroom

Present the data, and graphs to the class. Go over the methods in which the ihe is thrown, how the units were converted, and how the angle was calculated.

ASSESSMENT

Common Core State Standards for Mathematics	Skills	Concepts	Assessment
A.REI.1 - Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	Using logical reasoning to come up with a method to solve for the unknown quantities.	Logical Reasoning	Group/individual demonstration of the entire project to teachers and peers
F.IF.7 - Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.	Use excel to display data and create graphs that allow students to make inferences	Technological components	Look over the tables and graphs of the students to check for accuracy
F.TF.7 - Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	Use trigonometric functions and methods to find solutions so that students can make inferences about them.	Determine values of angles given the two lengths of the sides.	Feedback from students and teacher on how the desired measurements were calculated.

Ola Pathways	Skills	Concepts
NHMO.1 – Incorporate cultural traditions, language, history, and values in meaningful holistic prcsesses to nourish the emotional, physical, mental/intellectual, social, and spiritual well-being of the learning community that promote healthy mauli and mana.	Using Hawaiian Anthropic units and unit conversions to measure distances.	Hawaiian Anthropic Units.
NHMO.3 – Sustain respect for the integrity of one's own cultural knowledge and provide meaningful opportunities to make new connections among other knowledge systems.		Makahiki and games played during the Makahiki.



One of the two reference ropes.
Key:
Orange: Malua — **Red**: Kikoʻo
Purple: Maha — **Green**: Haʻilima

Picture by Geuseppe Ayala

OBSERVATION

Notes, Observations, Future Recommendations

- Sometimes after being thrown, the ihe will roll. Do you make the students take this to consideration or do they re-throw?
- Because the rope will be made off of your measurements and be marked by you, there will be errors in the spacing of each marker. This could lead to significant errors.
- Also, if the rope is placed and it is stretched the spacing of the marks could be inflated.
- Possibly have your students mark the ropes using their own measurements.
- Have them collect data when throwing overhand and underhand.
- Possibly come up with another design for the ihe. This is because the broom handle inside of the silly noodle may shift during flight and impact.
- Possibly let the students try to find the standard deviation from the reference rope.

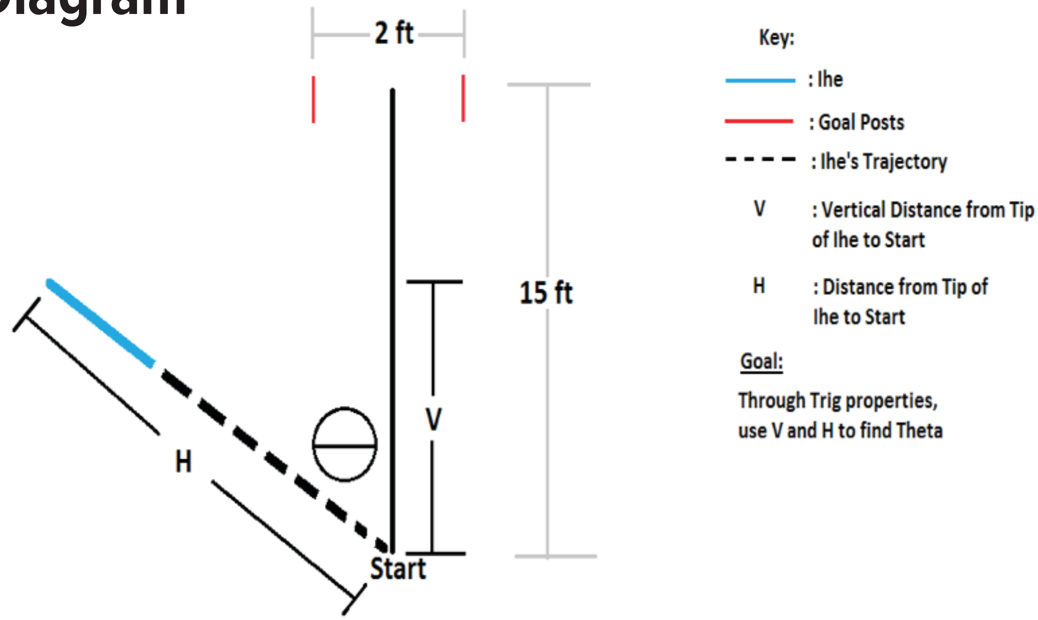
References

- Recreation Games. (2014). *Games of Skill*. Retrieved from http://www.hawaiihistory.org/index.cfm?fuseaction=ig.page&PageID=520
- Early Hawaiian Society. (2005-2011). *Sports and Games.* Retrieved from http://www.hawaiialive.org/topics.php?sub=Early+Hawaiian+Society&Subtopic=110
- Be Your own Laʻau Ana. Dey, Darienne. *Ethnomathematics & STEM Institute: Professional Development Workshop #2*. January 2014

EXAMPLE

Example Lesson

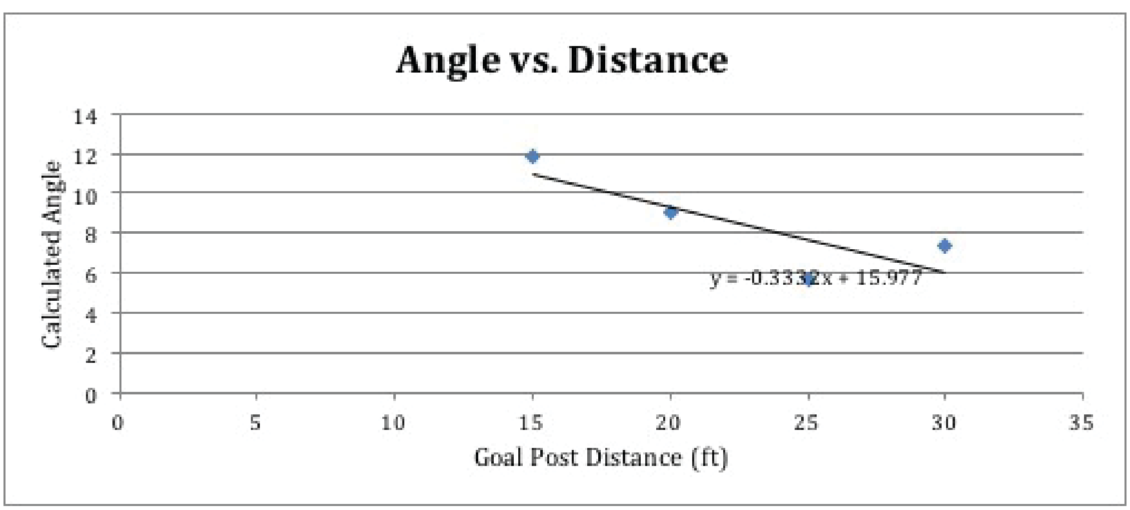
Diagram



Student 1 Data Example

Goal Post Distance	Vertical Distance (Hawaiian)	Vertical Distance (English)	Hypotenuse Distance (Hawaiian)	Hypotenuse Distance (English)	Calculated Angle	Measured Angle
15 ft.	23.25 Kikoʻo	139.5 in	23.75 Kikoʻo	142.5 in	11.78	10
20 ft.	39.25 Kikoʻo	235.5 in	39.75 Kikoʻo	238.5 in	9.09	13
25 ft.	50.50 Kikoʻo	303 in	50.75 Kikoʻo	304.5 in	5.69	9
30 ft.	60.25 Kikoʻo	361.25 in	60.75 Kikoʻo	364.5 in	7.36	6

Student 1 Graph Example



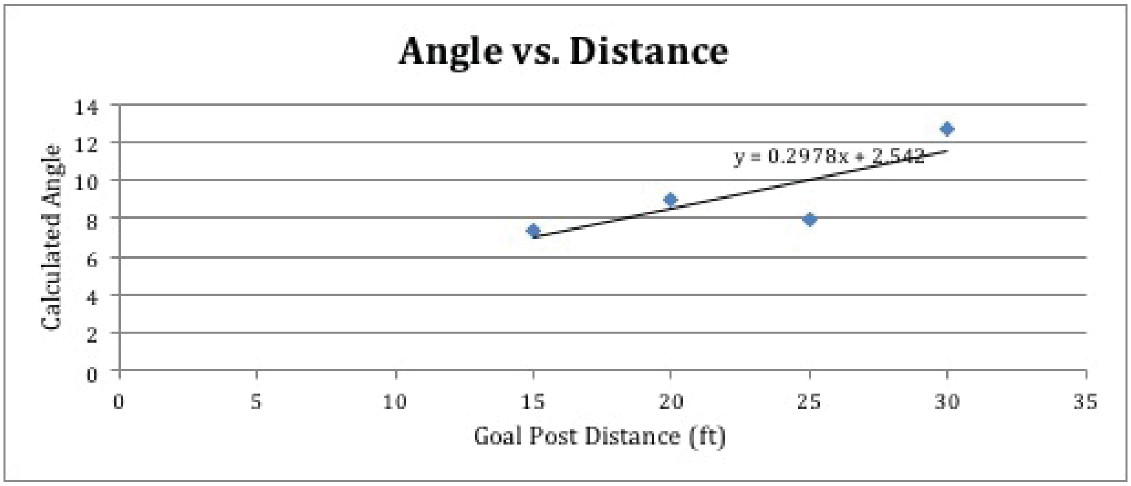
Hawaiian Anthropic Units

- **ʻowā** = ½ finger's width = 3/8 in
- **mākahi** = 1 finger's width = ¾ in
- **mālua** = 2 fingers' width = 1 ½ in = .25 kīkoʻo
- **mākolu** = 3 fingers' width = 2 ¼ in
- **māhā** = 4 fingers' width = 3 in = .5 kīkoʻo
- **kīkoʻo** = tip of pointer finger to tip of thumb (hypotenuse) = 6 in
- **anana** = wingspan = 69 in = 5 ft 9 in
- **pīʻā** = length of hand (tip of middle finger to heel of hand) = 7 ¼ in
- **haʻilima** = elbow to fingertips = 18 in = 1 ft 6 in = 3 kīkoʻo
- **iwilei** = collarbone to fingertips = 34 in = 2 ft 10 in
- **muku** = other elbow to fingertips = 50 in = 4 ft 2 in

Student 2 Data Example

Goal Post Distance	Vertical Distance (Hawaiian)	Vertical Distance (English)	Hypotenuse Distance (Hawaiian)	Hypotenuse Distance (English)	Calculated Angle	Measured Angle
15 ft.	30.25 Kikoʻo	181.5 in	30.5 Kikoʻo	183 in	7.34	5
20 ft.	40 Kikoʻo	240 in	40.5 Kikoʻo	243 in	9.01	15
25 ft.	51.25 Kikoʻo	307.5 in	51.75 Kikoʻo	310.5 in	7.97	14
30 ft.	60.50 Kikoʻo	363 in	61. 75 Kikoʻo	370.5 in	12.65	17

Student 2 Graph Example



Analysis and Conclusion

From the data, we see that the slope (m1) of the line y = -0.3332x + 15.977 is -0.3332. Since m1 is negative, we take the absolute value |m1| = |-0.3332| = 0.3332. We also see the slope (m2) of the line y = 0.2978x + 2.542 is 0.2978, so m2 = 0.2978.

The slope of the lines is expressed as angle/ distance, this would mean that the lower the slope of the line, the more accurate the thrower is. Since m2 < m1, we can say that Student 2 is the more skilled thrower than Student 1. However, since Student 1's line has negative slope and Student 2's line has positive slope, we can say that Student 1 got more accurate the farther the distances got and Student 2 got less accurate the father the distances got.