
Photo by stillwellmike

Resources and Materials

- Promethean Board and Laptop for notes, Stellarium, and to record class discussion topics
- Star trail photos
- Protractors
- Compass
- Pencils
- Paper

Key Definitions

- Circle
- Arc
- Major Arc
- Minor Arc
- Arc Length
- Central Angle
- Subtend

Objectives

The students will:

- Discover major arcs of circles, minor arcs of circles, semicircles, central angles of circles and their arcs as well as each of their measures.
- Calculate measures of central angles.
- Calculate measures of arcs.

DISCOVERING CENTRAL ANGLES AND ARCS USING TRADITIONAL POLYNESIAN NAVIGATION AND STAR TRAILS

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9th - 12th Grade / High School
Geometry

Overview

In this lesson students will discover central angles and their arcs through the use of the importance of stars for traditional Polynesian navigation. Students will be given a background of Hawaiian star lines and the names for these stars. They will look for geometric shapes and patterns in the Hawaiian Star lines of Na Ohana Hoku Eha (The Four Star Families). We will discuss the movement of stars with the use of Stellarium and pictures of star trails. Students will then work in groups to see how far one individual star traveled in one of the star trail pictures. In their groups they should brainstorm what they already know about circles and how to approach this problem. Each group will have protractors and compasses. They will need to trace or redraw the circle with the arc and radii. Together we will discover that central angles are equal to their subtended arcs (in degrees).

Standards

CCSS.MATH.CONTENT.HSG.C.A.2

Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.

Additionally, it addresses the Common Core Standards for Mathematical Practices:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

LESSON

Adaptations

The students have recently been learning about circles, their parts, areas, and circumference. The class will be expanding upon this knowledge by learning about arc length and central angles and connecting these concepts to real world applications. The teacher will be walking around to assist students and provide support by asking them questions that promote critical thinking. Extra time will be allotted to students who need it.

Opening

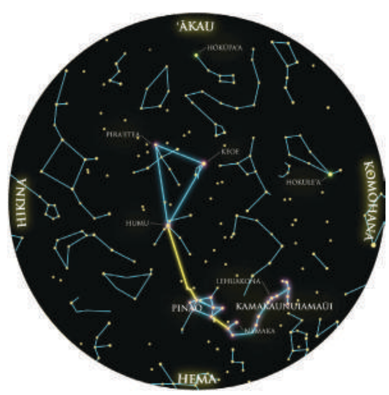
The teacher will ask the students what they know of Hokulea's Worldwide Voyage and how they and our ancestors navigated without modern technology. The teacher will give the students a background of Hawaiian star lines and the names for these stars. We will discuss the importance of stars in navigation.

The class will make observations of patterns and geometric shapes created by stars in the sky that Nainoa Thompson has organized into four star lines in the Hawaiian Star lines of Na Ohana Hoku Eha (The Four Star Families).

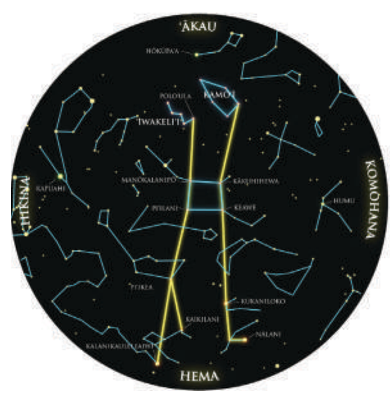
Na Ohana Hoku Eha

Kekaomakaliʻi
("The Canoe-Bailer of Makaliʻi")

Kaiwikuamoʻo
("The Backbone")



Manaiakalani
("Maui's Fishhook")



Kalupeakawelo
("The Kite of Kawelo")

**Images from ʻImiloa*

Photo courtesy of Ashley Deeks

LESSON

One way to remember the sequence of the four quadrants of the sky is to remember it geometrically:

1. A ***curve*** (Ke ka, "the Canoe-Bailer"); followed by
2. A ***line*** ("Kaiwikuamoʻo); followed by
3. Manaiakalani contains the three bright stars of the Navigator's ***triangle***; followed by
4. A ***square*** (the fourth quarter of the sky includes the Great Square of Pegasus).

The class will discuss how wayfinders memorize the position of stars on the celestial sphere in order to use them as directional clues when they rise and set. That even on cloudy nights, wayfinders can recognize isolated stars or star groups and imagine the rest of the celestial sphere around them.

The class will discuss how wayfinders can still find their way as the stars move across the sky and the movement of stars with the use of Stellarium and pictures of star trails.

Essential Questions

- How are the essential components/parts of a circle interconnected/organized?
- What is the relationship between the central angle and its subtended arc?
- How do you determine the measure of a central angle and its subtended arc?

The students will find photos online of other star maps or the teacher can provide photos for them.

Students will then work in groups to see how far one individual star traveled. In their groups they should brainstorm what they already know about circles, which they should have just learned. Each group will have protractors and compasses.

Angle & Arc Observation

Since the students had just learned about circles they should remember that a circle has 360 degrees. Also that the diameter creates an angle which is 180 degrees. Since we know that half of a circle is 180 degrees then we know that a central angle corresponds directly with the arc it creates.

LESSON

Students will then work in groups to see how far one individual star traveled. In their groups they should brainstorm what they already know about circles, which they should have just learned. Each group will have protractors and compasses.

After a little while the class will discuss what each group discovered. The students should discover that the distance traveled by the star is a section of a circle. The teacher will then define to them what an arc is. They will again go back to their groups and discuss this new information and draw this circle on a separate paper with the designated arc that they are trying to find with its radii.

Probing Questions

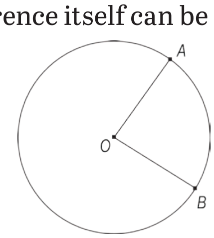
- How many degrees are there in a circle?
- What happens as we extend one radius to a diameter?
- Does the diameter contain an arc?
- How many degrees would the arc created by that diameter contain?
- Is there a relationship between the degrees of the diameter and the arc it creates?

The students will discover the relationship between a central angle and the arc that it creates by examining the relationship between the diameter and its subtended arc. The teacher will then define a central angle to the class and the relationship between the central angle and the arch that it creates. Then the students will only have to find the measure of the central angle to find the measure of the arc (the distance in which the star traveled). They will also find at least 3 more measures of star trails.

As an extension one could teach the students how to convert degrees to radians.

An ***arc*** of a circle is a "portion" of the circumference of the circle.

The ***length of an arc*** is simply the length of its "portion" of the circumference. Actually the circumference itself can be considered an arc length.



The length of an arc or arc length is traditionally symbolized by "$\widehat{AB}$".

Central angles are angles formed by any two radii in a circle. The vertex is the center of the circle. In the diagram above $\widehat{AB}$ is the arc to the central angle, $\angle AOB$.

The central angle equals the arc it subtends.

Example: $\angle AOB = \widehat{AB}$

Subtend- To be opposite to.

Arcs are measured in three different ways. They are measured in degrees and in unit length as follows:

- ***Degree measure of a semicircle:*** This is 180°. Its unit length is half of the circumference of the circle.
- ***Degree measure of a minor arc:*** Defined as the same as the measure of its corresponding central angle. Its unit length is a portion of the circumference. Its length is always less than half of the circumference.
- ***Degree measure of a major arc:*** This is 360° minus the degree measure of the minor arc that has the same endpoints as the major arc. Its unit length is a portion of the circumference and is always more than half of the circumference.

Students will be evaluated by their participation in the activity and by their reflections of the activity at the end of class.

Grading Rubric:

CATEGORY	4	3	2	1
Collaboration	Student is an engaged partner, listening to suggestions of others and working cooperatively throughout lesson.	Student is an engaged partner but had trouble listening to others and/or working cooperatively.	Student cooperates with others, but needs prompting to stay on-task.	Student does not work effectively with others.
Reflection	Reflection is thoughtful, detailed and clear.	Reflection is thoughtful, and clear.	Reflection is a little difficult to under-stand, but includes critical components.	Reflection is difficult to understand and is missing several components OR was not included.
Strategy/ Procedures	Typically, uses an efficient and effective strategy to solve the problem(s).	Typically, uses an effective strategy to solve the problem(s).	Sometimes uses an effective strategy to solve problems, but does not do it consistently.	Rarely uses an effective strategy to solve problems.
Use of Manipulatives	Student always listens to and follows directions and only uses manipulatives as instructed.	Student typically listens to and follows directions and mostly uses manipulatives as instructed.	Student sometimes listens to and follows directions and uses manipulatives appropriately when reminded.	Student rarely listens and often "plays" with the manipulatives instead of using them as instructed.

Cultural and Place Based Learning

A field trip to the Planetarium at the Bishop museum or the students could take their own star trail photos and find arc lengths. Also a field trip to the Polynesian Voyaging Society could be an extension of this lesson. The activities they do with the star compass could easily be integrated to math, especially geometry with transformations, circles, angles, etc.

Also following the Hōkūleʻa's World Wide Journey and communicating with the crewmembers could be extremely beneficial for the students with not only their comprehension and appreciation of mathematics but of the culture of Hawaii.